Published by Institute of Physics Publishing for SISSA/ISAS
Received: *January 27, 2004*
Accepted: *February 21, 2004*

# Type IIB tensionless superstrings in a pp-wave background

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Abstract: We solve the tensionless string in a constant plane wave background and obtain a hugely degenerate spectrum. This is the case for a large class of plane wave backgrounds. We show that the solution can also be derived as a consistent limit of the quantized tensile theory of IIB strings in a pp-wave. This is in contrast to the situation for several other backgrounds.

Keywords: Superstrings and Heterotic Strings, Penrose limit and pp-wave background.


http://jhep.sissa.it/archive/papers/jhep022004051/jhep022004051.pdf

JHEP02(2004)051

## Contents



## 1. Introduction

Although the study of tensionless strings has a long history [1]–[17], it has become of more wide-spread interest only recently. Originally conceived as a limit of the same type as that of massless relativistic particles and of similar interest for the study of high-energy behavior, it is presently perhaps studied mainly for its possible relations to higher spin theory [18]–[20].

Interest in the relation to higher spin theory was triggered by the observation that such a spectrum seems to arise on the Yang-Mills side in the AdS/CFT correspondence [21, 22], where the limit $T \to 0$ corresponds to free Yang-Mills. To really corroborate the correspondence in this limit, one should display the spectrum of tensionless strings in the bulk, i.e. in AdS space. However, quantizing the classical tensionless string in that space meets with similar problems as does quantizing the tensile string [23, 24].[1] A more modest ambition which we settle for here is to study the spectrum of tensionless strings in a Penrose limit of $AdS_5 \times S^5$, i.e. in a maximally supersymmetric pp-wave background [26, 27].

[1] See, however, [25].

In this paper we first quantize the classical tensionless (super)string in the relevant pp-wave background and study its spectrum. We find a hugely degenerate spectrum which may be classified in terms of affine Lie-algebras. Then, we move on to study the quantized tensile (super)string first discussed by Metsaev and Tseytlin [28, 29],[2] recovering the same spectrum (and the same model) in a certain well-defined limit.

Apart from shedding light on the behavior of quantized tensionless strings in this kind of curved background, our results also illuminate the long-standing question of whether the tensionless limit commutes with quantization: In this case it does. At the same it has been shown that this is not the case, e.g. for strings moving in backgrounds that are only approximate string solutions [31]. We would argue that the reason quantization commutes with the tensionless limit in our case is that the pp-wave background we consider is a solution to all order in $\alpha'$. This argument might seem to contradict the difficulties in showing commutation in a flat background. An important difference between these two cases, however, is the existence of a mass scale in the pp-wave to which the tension may be compared. It is also this mass scale which seems to allow a straight-forward generalization of our results to homogeneous plane waves. These are backgrounds slightly more complicated than the one we study and have recently been discussed [32, 33].

Finally, we note that the tension and mass scale always appear in such a combination that the tensionless limit is equivalent to the mass becoming infinitely large, corresponding to an infinite background curvature.

The paper is organized as follows: In section 2, we derive the action for the tensionless IIB string in the (maximally supersymmetric) constant plane wave background starting from the corresponding results in the tensile case. Section 3 contains the classical and quantized solutions to this model and derive the spectrum. In section 4, we obtain our previous results as a well-defined limit of the tensile string solutions in this background. Section 5 deals with the homogeneous plane waves. The paper is concluded with a short discussion and outlook in section 6.

## 2. The action for a tensionless superstring in a pp-wave background

In this section, we derive the tensionless superstring action in the constant plane wave background. This is a special plane wave with a constant Ramond-Ramond 5-form field strength given by[3]

$$\mathrm{d}s^2 = 2\mathrm{d}x^+\mathrm{d}x^- - \mu^2 x^I x_I \mathrm{d}x^+\mathrm{d}x^+ + \mathrm{d}x^I\mathrm{d}x_I, \qquad F_{+1234} = F_{+5678} = 2\mu\,. \tag{2.1}$$

The type IIB superstring in this background was first discussed by Metsaev [28] and then solved by Metsaev and Tseytlin [29]. For simplicity, our notation closely follows that of the latter paper. A short overview can be found in appendix A. Below, we derive

[2] A good introductory review can be found in [30].

[3] $I = 1...8$ denote the eight transverse coordinates while $x^\pm = \frac{1}{\sqrt{2}}(x^9 \pm x^0)$ are the usual light-cone coordinates.



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the corresponding light-cone lagrangian for a tensionless superstring starting from the $\kappa$-symmetry fixed lagrangian given in [28]:

$$\mathcal{L} = -\frac{T}{2}\sqrt{-g}g^{ab}h_{ab} + \mathrm{i}T\epsilon^{ab}\partial_a X^+\left(\theta^1\bar{\gamma}^-\partial_b\theta^1 - \theta^2\bar{\gamma}^-\partial_b\theta^2\right). \tag{2.2}$$

Here, $T$ is the string tension and

$$\begin{aligned} h_{ab} &= 2\partial_a X^+\left(\partial_b X^- + \mathrm{i}\theta^1\bar{\gamma}^-\partial_b\theta^1 + \mathrm{i}\theta^2\bar{\gamma}^-\partial_b\theta^2\right) - \\ &\quad - \left(\mu^2 X^I X_I + 4\mathrm{i}\mu\theta^1\bar{\gamma}^-\Pi\theta^2\right)\partial_a X^+\partial_b X^+ + \partial_a X^I\partial_b X_I\,. \end{aligned} \tag{2.3}$$

To be able to take the tension $T$ to zero, we go to the phase space lagrangian in the usual way [8]:

$$\begin{aligned} \mathcal{L}_{ps} &= \mathrm{i}\mathcal{P}_-\left(\theta^1\bar{\gamma}^-\dot{\theta}^1 + \theta^2\bar{\gamma}^-\dot{\theta}^2\right) - \mathrm{i}TX'^+\left(\theta^1\bar{\gamma}^-\dot{\theta}^1 - \theta^2\bar{\gamma}^-\dot{\theta}^2\right) + \mathcal{P}_+\dot{X}^+ + \mathcal{P}_-\dot{X}^- + \mathcal{P}_I\dot{X}^I - \\ &\quad -\varphi\Big(2\mathcal{P}_+\mathcal{P}_- + \left(\mu^2X^IX_I + 4\mathrm{i}\mu\theta^1\bar{\gamma}^-\Pi\theta^2\right)\mathcal{P}_-^2 + \\ &\qquad + \mathcal{P}_I\mathcal{P}^I + 2\mathrm{i}T(-\mathcal{P}_- + TX'^-)\theta^1\bar{\gamma}^-\theta'^1 + 2\mathrm{i}T(\mathcal{P}_- + TX'^-)\theta^2\bar{\gamma}^-\theta'^2 + \\ &\qquad + 2T^2X'^+X'^- - T^2\left(\mu^2X^IX_I + 4\mathrm{i}\mu\theta^1\bar{\gamma}^-\Pi\theta^2\right)(X'^+)^2 + T^2X'_IX'^I\Big) - \\ &\quad - \rho\left(\mathcal{P}_+X'^+ + \mathcal{P}_-X'^- + \mathcal{P}_IX'^I + \mathrm{i}(\mathcal{P}_- - TX'^+)\theta^1\bar{\gamma}^-\theta'^1 + \mathrm{i}(\mathcal{P}_- + TX'^+)\theta^2\bar{\gamma}^-\theta'^2\right). \end{aligned}$$

Herein, $\varphi$ and $\rho$ are Lagrange multipliers multiplying the constraints on the canonical momenta $\mathcal{P}_\mu$. Taking the limit $T \to 0$ in the phase space lagrangian presents no problem [8]. With the identification $V^a = \frac{1}{2\sqrt{\varphi}}(1, -\rho)$, we obtain

$$\begin{aligned} \mathcal{L}_0 &= V^aV^b\Big(2\partial_aX^+\left(\partial_bX^- + \mathrm{i}(\theta^1\bar{\gamma}^-\partial_b\theta^1 + \theta^2\bar{\gamma}^-\partial_b\theta^2)\right) - \\ &\quad - \left(\mu^2X^IX_I + 4\mathrm{i}\mu\theta^1\bar{\gamma}^-\Pi\theta^2\right)\partial_aX^+\partial_bX^+ + \partial_aX^I\partial_bX_I\Big). \end{aligned} \tag{2.4}$$

The corresponding action is invariant under world-sheet diffeomorphisms with $V^a$ transforming as a vector density

$$\begin{aligned} \delta X^\mu &= \epsilon^a\partial_aX^\mu \\ \delta\theta^\alpha &= \epsilon^a\partial_a\theta^\alpha \\ \delta V^a &= -V^b\partial_b\epsilon^a + \epsilon^b\partial_bV^a + \frac{1}{2}\partial_b\epsilon^bV^a\,. \end{aligned} \tag{2.5}$$

This symmetry allows us to fix the gauge $V^a = (v, 0)$ with $v$ a (dimensionful) constant. The lagrangian then becomes

$$\mathcal{L}_0 = v^2\Big(2\dot{X}^+\dot{X}^- + \dot{X}^I\dot{X}_I - \left(\mu^2X^IX_I + 4\mathrm{i}\mu\theta^1\bar{\gamma}^-\Pi\theta^2\right)(\dot{X}^+)^2 + 2\mathrm{i}\dot{X}^+(\theta^1\bar{\gamma}^-\dot{\theta}^1 + \theta^2\bar{\gamma}^-\dot{\theta}^2)\Big), \tag{2.6}$$

which now has to be supplemented by the constraints coming from a variation of the action with respect to $V^a$. As in the tensile case there is still a residual symmetry left after this gauge fixing. The symmetry transformations are

$$\delta\tau = f'(\sigma)\tau + g(\sigma)\,, \qquad \delta\sigma = f(\sigma) \tag{2.7}$$



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with $f$ and $g$ arbitrary smooth and differentiable functions of $\sigma$. This allows us to go to light-cone gauge by choosing $X^+ = \frac{p^+}{v^2}\tau$. Absorbing the dependence on $v$ and $p^+$ into the fields according to $X^I \to v^{-1}X^I$ and $\theta^\alpha \to (p^+)^{-1/2}\theta^\alpha$ we obtain the light-cone lagrangian of the tensionless superstring in the pp-wave background eq. (2.1)

$$\mathcal{L}_0 = \dot{X}^I\dot{X}_I - m^2 X^I X_I + 2\mathrm{i}\left(\theta^1\bar{\gamma}^-\dot{\theta}^1 + \theta^2\bar{\gamma}^-\dot{\theta}^2\right) - 4\mathrm{i}m\theta^1\bar{\gamma}^-\Pi\theta^2\,, \tag{2.8}$$

where $m = \frac{\mu p^+}{v^2}$. The fields $X^I$ and $\theta$ are now dimensionless while $X^+$ and $X^-$ still have the dimension of length.

## 3. Quantized solution of the model

In this section we find classical solutions to and quantize the tensionless model eq. (2.8). We describe the quantization procedure and present the spectrum.

### 3.1 Solution of the classical equations of motion

The equations of motion for the action corresponding to eq. (2.8) are

$$\ddot{X}^I + m^2 X^I = 0\,, \tag{3.1}$$

$$\dot{\theta}^1 - m\Pi\theta^2 = 0\,, \tag{3.2}$$

$$\dot{\theta}^2 + m\Pi\theta^1 = 0\,. \tag{3.3}$$

Varying eq. (2.4) with respect to $V^a$ gives the respective "Virasoro constraints":

$$0 = 2p^+\dot{X}^- - m^2X^IX_I + \dot{X}^I\dot{X}_I + 2\mathrm{i}\left(\theta^1\bar{\gamma}^-\dot{\theta}^1 + \theta^2\bar{\gamma}^-\dot{\theta}^2 - 2m\theta^1\bar{\gamma}^-\Pi\theta^2\right) \tag{3.4}$$

$$0 = p^+X'^- + X'^I\dot{X}_I + \mathrm{i}\left(\theta^1\bar{\gamma}^-\theta'^1 + \theta^2\bar{\gamma}^-\theta'^2\right). \tag{3.5}$$

The part containing the fermion fields in eq. (3.4) is identically zero due to the equations of motion. Solving (3.1)–(3.3) for closed string boundary conditions results in the following expansion of the transverse fields:

$$\begin{aligned} X^I(\sigma,\tau) &= \cos(m\tau)x_0^I + m^{-1}\sin(m\tau)p_0^I + \\ &+ \mathrm{i}m^{-1}\sum_{n\neq 0}\mathrm{sign}(n)\left\{\alpha_n^{1I}\mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\tau - 2n\sigma)} + \alpha_n^{2I}\mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\tau + 2n\sigma)}\right\}, \end{aligned} \tag{3.6}$$

$$\begin{aligned} \theta^1(\sigma,\tau) &= \cos(m\tau)\theta_0^1 + \sin(m\tau)\Pi\theta_0^2 + \\ &+ \frac{1}{\sqrt{2}}\sum_{n\neq 0}\left\{\theta_n^1\mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\tau - 2n\sigma)} + \mathrm{i}\Pi\theta_n^2\mathrm{sign}(n)\mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\tau + 2n\sigma)}\right\}, \end{aligned} \tag{3.7}$$

$$\begin{aligned} \theta^2(\sigma,\tau) &= \cos(m\tau)\theta_0^2 - \sin(m\tau)\Pi\theta_0^1 + \\ &+ \frac{1}{\sqrt{2}}\sum_{n\neq 0}\left\{\theta_n^2\mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\tau + 2n\sigma)} - \mathrm{i}\Pi\theta_n^1\mathrm{sign}(n)\mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\tau - 2n\sigma)}\right\}. \end{aligned} \tag{3.8}$$

Despite the fact that the frequencies become degenerate and, moreover, collapse to only one frequency, these solutions look like those derived in the tensile case [29]. Since we consider closed string boundary conditions, we may integrate the constraint eq. (3.5)

$$0 = p^+\left(X^-(\sigma=\pi) - X^-(\sigma=0)\right) + \int_0^\pi \mathrm{d}\sigma\left\{\dot{X}^IX_I' + \mathrm{i}\left(\theta^1\bar{\gamma}^-\theta'^1 + \theta^2\bar{\gamma}^-\theta'^2\right)\right\}, \tag{3.9}$$

and drop the first term, because it is zero. In terms of the mode expansion, the constraint reads

$$N^1 = N^2\,, \qquad N^{\mathcal{I}} \equiv 2\pi \sum_{n\neq 0} \left\{ \frac{n}{m}\,\mathrm{sign}(n)\,\alpha^{\mathcal{I}}_{-n}\cdot\alpha^{\mathcal{I}}_{n} + n\,\theta^{\mathcal{I}}_{-n}\bar{\gamma}^-\theta^{\mathcal{I}}_{n} \right\}, \tag{3.10}$$

with $\mathcal{I} = 1, 2$. This is the usual level-matching condition.

### 3.2 Light-cone hamiltonian

The light-cone hamiltonian is derived by integrating the momentum conjugate to the time coordinate $X^+$, i.e.

$$H_{lc} = -\int \mathrm{d}\sigma\,\mathcal{P}^-\,. \tag{3.11}$$

In the diffeomorphism and light-cone gauge and by using (3.2)–(3.4) it can be written as:

$$H_{lc} = \frac{v^2}{p^+}\int \mathrm{d}\sigma \left\{ \dot{X}^I\dot{X}_I + m^2 X^I X_I + 2\mathrm{i}(\theta^1\bar{\gamma}^-\dot{\theta}^1 + \theta^2\bar{\gamma}^-\dot{\theta}^2) \right\}. \tag{3.12}$$

In terms of the mode expansions of the fields we find

$$\begin{aligned} H_{lc} = {} & \frac{\pi v^2}{p^+}\left(p_0^2 + m^2 x_0^2\right) + 4\pi\mathrm{i}\frac{m v^2}{p^+}\theta_0^1\bar{\gamma}^-\Pi\theta_0^2 + \\ & + \frac{2\pi v^2}{p^+}\sum_{n\neq 0}\left\{\alpha^1_{-n}\cdot\alpha^1_n + \alpha^2_{-n}\cdot\alpha^2_n + m\,\mathrm{sign}(n)\left(\theta^1_{-n}\bar{\gamma}^-\theta^1_n + \theta^2_{-n}\bar{\gamma}^-\theta^2_n\right)\right\}. \end{aligned} \tag{3.13}$$

This light-cone hamiltonian gives the energy for the classical states.

### 3.3 Canonical quantization

To obtain the corresponding quantum theory, we apply the equal time canonical commutation relations to the fields $X^I$ and their conjugate momenta $\mathcal{P}^I = 2\dot{X}^I$:

$$\left[\mathcal{P}^I(\sigma,\tau), X^J(\sigma',\tau)\right] = -\mathrm{i}\delta^{IJ}\delta(\sigma-\sigma')\,. \tag{3.14}$$

These translate into commutation relations for the modes as follows

$$\left[p_0^I, x_0^J\right] = -\frac{\mathrm{i}}{2\pi}\delta^{IJ}\,, \tag{3.15}$$

$$\left[\alpha_n^{\mathcal{I}I}, \alpha_{n'}^{\mathcal{J}J}\right] = \frac{m}{4\pi}\,\mathrm{sign}(n)\,\delta_{n+n',0}\,\delta^{IJ}\delta^{\mathcal{I}\mathcal{J}}\,,\ \ n,n' \in \mathbb{Z}\backslash\{0\}\,. \tag{3.16}$$

For the fermionic fields we have a second class constraint, since the momenta for these fields, $-2\mathrm{i}\bar{\gamma}^-\theta^{\mathcal{I}}$, are linearly dependent on the fields themselves. This yields the anti-commutation relations

$$\left\{\theta^{\mathcal{I}\alpha}(\sigma,\tau), \theta^{\mathcal{J}\beta}(\sigma',\tau)\right\} = \frac{1}{8\pi}(\gamma^+)^{\alpha\beta}\delta^{\mathcal{I}\mathcal{J}}\delta(\sigma-\sigma')\,, \tag{3.17}$$

which for the modes results in

$$\left\{\theta_n^{\mathcal{I}\alpha}, \theta_{n'}^{\mathcal{J}\beta}\right\} = \frac{1}{8\pi}(\gamma^+)^{\alpha\beta}\delta^{\mathcal{I}\mathcal{J}}\delta_{n+n',0}\,,\ \ n,n' \in \mathbb{Z}\,. \tag{3.18}$$

Equations (3.16), (3.18) imply that we can promote the bosonic and fermionic negative frequency modes to creation operators and the corresponding positive ones to annihilation operators. To define a proper vacuum state, we collect the zero-modes into harmonic oscillator modes

$$\alpha_0^I = \frac{m}{2}\left(x_0^I + \frac{\mathrm{i}}{m}p_0^I\right), \qquad \alpha_0^{\dagger I} = \frac{m}{2}\left(x_0^I - \frac{\mathrm{i}}{m}p_0^I\right), \tag{3.19}$$

obeying the commutation relation

$$\left[\alpha_0^I, \alpha_0^{\dagger J}\right] = \frac{m}{4\pi}\delta^{IJ}\,. \tag{3.20}$$

For the vacuum state we require that

$$\alpha_n^{\mathcal{I}}\left|0;p^+\right\rangle = 0,\ n \geq 0\,, \qquad \theta_n^{\mathcal{I}}\left|0;p^+\right\rangle = 0,\ n > 0\,. \tag{3.21}$$

While there is a unique choice for the bosonic part of the vacuum state, there is a variety of four possibilities for choosing the fermionic part [29], but in the end all of the latter are equivalent by a redefinition of the $\theta$-modes. This will be of no importance for what follows, our analysis can indeed be carried out for any of the possible choices of the fermionic vacuum states. The light-cone hamiltonian of the quantized theory can be obtained by using the above commutation relations to normal-order eq. (3.13):

$$\begin{aligned} H_{lc} = {} & \frac{4\pi v^2}{p^+}\alpha_0^{\dagger I}\alpha_0^J\delta_{IJ} + 4\pi\mathrm{i}\frac{mv^2}{p^+}\theta_0^1\bar{\gamma}^-\Pi\theta_0^2 + \\ & + \frac{4\pi v^2}{p^+}\sum_{n>0}\left\{\alpha_{-n}^1\cdot\alpha_n^1 + \alpha_{-n}^2\cdot\alpha_n^2 + m\,\left(\theta_{-n}^1\bar{\gamma}^-\theta_n^1 + \theta_{-n}^2\bar{\gamma}^-\theta_n^2\right)\right\}, \end{aligned} \tag{3.22}$$

where we used that in $D = 10$ the spinor space is 16 dimensional and the $\kappa$-symmetry fixing condition to drop the term proportional to $\gamma^-\bar{\gamma}^+$. This result resembles the light-cone hamiltonian in [29] despite the frequency degeneracy already seen for the fields. This implies that the energy spectrum of the quantum theory is infinitely degenerate.

### 3.4 The spectrum and the space of states

Starting from the vacuum state eq. (3.21), we count the number of states obtained by exciting with the bosonic oscillator modes on every energy level. The states are written in terms of the oscillators that act on the vacuum.

For the first levels, we also give the corresponding representations under the SO(4) $\times$ SO$'$(4) symmetry (table 1). The spectrum built from the fermionic oscillators is given in table 2. It is further possible to build states out of both the bosonic and the fermionic oscillators, but we do not pursue this here.

### 3.5 Degeneracy of the bosonic spectrum at the second energy level

From table 1 one sees that the bosonic part of the spectrum at energy level 2 is generated by states of the form $T_{IJ}\alpha^{1I}_{-n}\alpha^{2J}_{-n}|0\rangle$, where $\alpha^{\mathcal{I}I}_{-0} \equiv \alpha^{\dagger I}_0$, $\alpha^{\mathcal{I}I}_0 \equiv \alpha^I_0$ and $T_{IJ}$ is some polarization tensor. This corresponds to exactly one excitation in the $n$-th pair of oscillators. Denoting these states by the quantum number $n$, i.e.

$$|n\rangle_{IJ} = \frac{2\pi}{m}\alpha^{1I}_{-n}\alpha^{2J}_{-n}|0;p^+\rangle\,, \tag{3.23}$$

| state | energy-level | number of states |
|---|---|---|
| $\mathbf{1}$ | 0 | $(1,1)$ |
| $\alpha_0^{\dagger i}\,,\ \alpha_0^{\dagger i'}$ | 1 | $(4,1)\oplus(1,4)$ |
| $\alpha_0^{\dagger i}\alpha_0^{\dagger j}$<br>$\alpha_0^{\dagger i'}\alpha_0^{\dagger j'}$<br>$\alpha_0^{\dagger i}\alpha_0^{\dagger i'}$<br>$\alpha_{-n}^{1i}\alpha_{-n}^{2j}$<br>$\alpha_{-n}^{1i'}\alpha_{-n}^{2j'}$<br>$\alpha_{-n}^{1i}\alpha_{-n}^{2i'}$ | 2 | $(1,1)\oplus(9,1)$<br>$(1,1)\oplus(1,9)$<br>$(4,4)$<br>$(1,1)\oplus(9,1)\oplus(3^+,1)\oplus(3^-,1),\ n\in\mathbb{N}$<br>$(1,1)\oplus(1,9)\oplus(1,3^+)\oplus(1,3^-),\ n\in\mathbb{N}$<br>$(4,4),\ n\in\mathbb{N}$ |
| $(\alpha_0^\dagger)^3$<br><br>$\alpha_0^\dagger\alpha^1_{-n}\alpha^2_{-n}$<br>$\alpha^1_{-n}\alpha^1_{-m}\alpha^2_{-(n+m)},\ \alpha^1\alpha^2\alpha^2$ | 3 | $120=(1,1)\oplus(19,1)\oplus(4,1)\oplus(4,9)\oplus$<br>$(9,4)\oplus(1,4)\oplus(1,19)\oplus(1,1)$<br>$512,\ n\in\mathbb{N}$<br>$288+288,\ n,m\in\mathbb{N}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |

**Table 1:** The spectrum built from the bosonic modes.

| state | energy | number of states |
|---|---|---|
| $\mathbf{1}$ | 0 | 1 |
| $\theta_0^{\mathcal{I}\alpha}$ | 1 | $2\cdot 16=32$ |
| $\theta_0^{\mathcal{I}\alpha}\theta_0^{\mathcal{J}\beta}$<br>$\epsilon_{\mathcal{I}\mathcal{J}}\theta_{-n}^{\mathcal{I}\alpha}\theta_{-n}^{\mathcal{J}\beta}$ | 2 | $496=256+240$<br>$256,\ n\in\mathbb{N}$ |
| $\theta_0^{\mathcal{I}\alpha}\epsilon_{\mathcal{J}\mathcal{K}}\theta_{-n}^{\mathcal{J}\beta}\theta_{-n}^{\mathcal{K}\gamma}$<br>$\epsilon_{\mathcal{I}\mathcal{J}}\theta_{-n}^{\mathcal{I}\alpha}\theta_{-m}^{\mathcal{I}\beta}\theta_{-(n+m)}^{\mathcal{J}\gamma}$ | 3 | $n\in\mathbb{N}$<br>$n,m\in\mathbb{N}$ |
| $\vdots$ | $\vdots$ | $\vdots$ |

**Table 2:** The spectrum generated by the fermionic modes.

we define two operators[4]

$$\mathcal{A}^+ = \left(\frac{2\pi}{m}\right)^2 \sum_{IJ}\sum_{n\geq 0}\sqrt{n+1}\,\alpha^{1I}_{-n-1}\alpha^{1I}_{n}\,\alpha^{2J}_{-n-1}\alpha^{2J}_{n}\cdot\left(1-\frac{1}{2}\delta_{n,0}\right)$$
$$\mathcal{A}^- = \left(\frac{2\pi}{m}\right)^2 \sum_{IJ}\sum_{n\geq 0}\sqrt{n}\,\alpha^{1I}_{-n+1}\alpha^{1I}_{n}\,\alpha^{2J}_{-n+1}\alpha^{2J}_{n}\,. \tag{3.24}$$

These operators commute to $\mathbb{1}$ on the states under consideration and act as usual harmonic oscillator raising and lowering operators:

$$\mathcal{A}^+|n\rangle_{IJ} = \sqrt{n+1}|n\rangle_{IJ}\,, \qquad \mathcal{A}^-|n\rangle_{IJ} = \sqrt{n}|n-1\rangle_{IJ}\,, \qquad \mathcal{A}^-|0\rangle_{IJ} = 0\,. \tag{3.25}$$

Since there exists only one set of zero-modes, $|0\rangle_{IJ}$ is automatically symmetrized. The harmonic oscillator hamiltonian is defined in the usual way as well, i.e. $\tilde{H} = \mathcal{A}^+\mathcal{A}^-$,

[4] The additional factor in $\mathcal{A}^+$ is necessary because there is only one set of zero-modes, while all higher oscillator modes come in pairs.

yielding $\tilde{H}|n\rangle = n|n\rangle$. This algebra obviously commutes with the light-cone hamiltonian $H_{lc}$ given in eq. (3.22). Thus its action does not change the energy but rather reflects the degeneracy of the states.

### 3.6 Degeneracy of the bosonic spectrum at energy level 3

The discussion for energy level 3 is carried out in the same way was done for level 2. Denoting the states by

$$|n, m\rangle^{\mathcal{I}}_{IJK} \equiv \alpha^{\mathcal{J}I}_{-n}\alpha^{\mathcal{J}J}_{-m}\alpha^{\mathcal{I}K}_{-(n+m)}|0; p^+\rangle\,, \qquad \mathcal{I} \neq \mathcal{J}\,, \tag{3.26}$$

we may write down the following series (for $n + m =$ even)

$$\begin{aligned} |n, n\rangle^1 &\to |n-1, n+1\rangle^1 \to \ldots \to |1, 2n-1\rangle^1 \\ &\to |0, 2n\rangle^{1=2} \to |1, 2n-1\rangle^2 \to \ldots \to |n, n\rangle^2\,, \end{aligned} \tag{3.27}$$

and correspondingly for odd $n + m$

$$\begin{aligned} |n-1, n\rangle^1 &\to |n-2, n+1\rangle^1 \to \ldots \to |1, 2n-2\rangle^1 \\ &\to |0, 2n-1\rangle^{1=2} \to |1, 2n-2\rangle^2 \to \ldots \to |n-1, n\rangle^2\,. \end{aligned} \tag{3.28}$$

It is easy to see that these states form integer spin representations of $su(2)$ of increasing level as $n + m$ gets bigger. But these are nothing but the different levels of an affine $\widehat{su}(2)$ algebra! Correspondingly, the Heisenberg algebra in the previous subsection is indeed an affine $\hat{u}(1)$.

### 3.7 Bosonic sector partition function

Generalizing the above results, we obtain a partition function $Z_E$ for each energy level $E$ by the recursive formula

$$\begin{aligned} Z_0(x, y) &= 1\,, \\ Z_E(x, y) &= Z_{E-1} + \sum_{k=1}^{E-1} q_k(x) q_{E-k}(y) = \sum_{mn} \alpha_{mn} x^m y^n\,, \end{aligned} \tag{3.29}$$

where $q_k(x) = \sum_{n=0}^{\infty} q_k^n x^n$ is the generating functional for the number of partitions of $n$ with exactly $k$ parts. The sub-series with coefficients $\alpha_{nn}$ then encodes the dimension of the $n$-th multiplet at energy level $E$. As $E \to \infty$, the partition function goes to a very simple limit, namely

$$Z_E(x, y) \to Z_\infty(x, y) = \frac{(xy)^{1/24}}{\eta(x)\eta(y)} - \frac{x^{1/24}}{\eta(x)} - \frac{y^{1/24}}{\eta(y)}\,, \tag{3.30}$$

where $\eta(x)$ is the Dedekind $\eta$-function. In this context, an interesting question is the effect of this huge degeneracy on the BMN correspondence [34]. In particular, vanishing tension is related to the weak-coupling limit of the corresponding conformal field theory. Another open question is the relation to higher spin field theories.

## 4. The tensionless string as a limit of ordinary string theory in a pp-wave background

In this section we reverse the order of the procedure discussed in section 3. We first recapitulate some details of the quantized tensile theory in the pp-wave background eq. (2.1) [29] and then take the limit $T \to 0$ within this setting in a certain way. We show that this limit recovers our previous results. The light-cone and $\kappa$-symmetry fixed lagrangian in the tensile case reads

$$\begin{aligned}\tilde{\mathcal{L}} &= \tilde{\mathcal{L}}_B + \tilde{\mathcal{L}}_F \\ &= (\partial_+ \tilde{X}^I \partial_- \tilde{X}_I - \tilde{m}^2 \tilde{X}_I^2) + 2\mathrm{i}(\tilde{\theta}^1 \bar{\gamma}^- \partial_+ \tilde{\theta}^1 + \tilde{\theta}^2 \bar{\gamma}^- \partial_- \tilde{\theta}^2 - 2\tilde{m}\tilde{\theta}^1 \bar{\gamma}^- \Pi \tilde{\theta}^2)\,. \end{aligned} \tag{4.1}$$

Here, tilde refers to quantities in the tensile theory to distinguish them from the "bare" tensionless quantities. As in the previous discussion, we absorb the string tension $T$ in a rescaling of the fields. The equations of motions yield the closed string solutions

$$\begin{aligned}\tilde{X}^I(\sigma,\tau) &= \cos(\tilde{m}\tau)\tilde{x}_0^I + \frac{1}{\tilde{m}}\sin(\tilde{m}\tau)\tilde{p}_0^I + \\ &\quad + \mathrm{i}\sum_{n\neq 0}\frac{1}{\tilde{\omega}_n}\left\{\tilde{\alpha}_n^{1I}\mathrm{e}^{-\mathrm{i}(\tilde{\omega}_n\tau - 2n\sigma)} + \tilde{\alpha}_n^{2I}\mathrm{e}^{-\mathrm{i}(\tilde{\omega}_n\tau + 2n\sigma)}\right\}, \end{aligned} \tag{4.2}$$

$$\begin{aligned}\tilde{\theta}^1(\sigma,\tau) &= \cos(\tilde{m}\tau)\tilde{\theta}_0^1 + \sin(\tilde{m}\tau)\Pi\tilde{\theta}_0^2 + \\ &\quad + \sum_{n\neq 0}\tilde{c}_n\left\{\tilde{\theta}_n^1\mathrm{e}^{-\mathrm{i}(\tilde{\omega}_n\tau - 2n\sigma)} + \mathrm{i}\Pi\tilde{\theta}_n^2\frac{\tilde{\omega}_n - 2n}{\tilde{m}}\mathrm{e}^{-\mathrm{i}(\tilde{\omega}_n\tau + 2n\sigma)}\right\}, \end{aligned} \tag{4.3}$$

and similar for $\tilde{\theta}^2$. Herein,

$$\tilde{\omega}_n = \mathrm{sign}(n)\sqrt{\tilde{m}^2 + 4n^2} \qquad \text{and} \qquad \tilde{c}_n = \frac{1}{\sqrt{1 + \left(\frac{\tilde{\omega}_n - 2n}{\tilde{m}}\right)^2}}\,. \tag{4.4}$$

In taking the tension to zero, we set $T = v^2\lambda$ and take the dimensionless quantity $\lambda \to 0$. This limit is unproblematic if accompanied by a rescaling of the world-sheet time $\tau \to \bar{\tau} = \frac{\tau}{\lambda}$ where $\bar{\tau}$ is kept fixed. This corresponds to keeping the light-cone coordinate $\tilde{X}^+$ fixed in this limit and results in the world-sheet becoming a null surface. We introduce the following set of redefinitions:

$$\begin{aligned} m &= \lambda\tilde{m} \\ \omega_n &= \lambda\tilde{\omega}_n = \mathrm{sign}(n)\sqrt{m^2 + 4\lambda^2 n^2} \\ x_0^I &= \frac{1}{\sqrt{\lambda}}\tilde{x}_0^I \\ p_0^I &= \sqrt{\lambda}\tilde{p}_0^I \\ \alpha_n^I &= \sqrt{\lambda}\tilde{\alpha}_n^I\,. \end{aligned} \tag{4.5}$$

The new oscillator modes are independent of the string tension. This implies

$$\begin{aligned}\tilde{X}^I(\sigma,\bar{\tau}) &= \sqrt{\lambda}\Bigg[\cos(m\bar{\tau})x_0^I + \frac{1}{m}\sin(m\bar{\tau})p_0^I + \\ &\quad + \mathrm{i}\sum_{n\neq 0}\frac{1}{\omega_n}\left\{\alpha_n^{1I}\mathrm{e}^{-\mathrm{i}(\omega_n\bar{\tau} - 2n\sigma)} + \alpha_n^{2I}\mathrm{e}^{-\mathrm{i}(\omega_n\bar{\tau} + 2n\sigma)}\right\}\Bigg]\,. \end{aligned} \tag{4.6}$$

Because we absorbed the string tension into the fields, we may now take $\lambda \to 0$ in a non-trivial way after introducing new coordinates $X^I(\sigma, \bar{\tau}) = \lambda^{-1/2}\tilde{X}^I(\sigma, \bar{\tau})$. Comparing to eq. (3.6) we find a perfect correspondence. Already at this stage (just looking at the bosonic part of the theory) this indicates that — in contrast to flat space — tensionless string theory in the pp-wave is an uncomplicated limit of the ordinary tensile one. To underpin this statement we take a closer look at the fermionic fields $\tilde{\theta}^1(\sigma, \tau)$ and $\tilde{\theta}^2(\sigma, \tau)$. The fermionic oscillator modes are independent of the string tension even in the tensile theory and thus, under $\lambda \to 0$, we may take $\tilde{\theta}_n^{\mathcal{I}\alpha} \to \theta_n^{\mathcal{I}\alpha}$. Furthermore, $c_n \to \frac{1}{\sqrt{2}}$, $\frac{\omega_n - 2\lambda n}{m} \to \mathrm{sign}(n)$ for all $n$. In this limit the fields become

$$\begin{aligned}\theta^1(\sigma, \bar{\tau}) &= \cos(m\bar{\tau})\theta_0^1 + \sin(m\bar{\tau})\Pi\theta_0^2 + \\ &+ \frac{1}{\sqrt{2}}\sum_{n\neq 0}\left\{\theta_n^1 \mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\bar{\tau} - 2n\sigma)} + \mathrm{i}\Pi\theta_n^2 \mathrm{sign}(n)\mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\bar{\tau} + 2n\sigma)}\right\}, \quad (4.7)\\ \theta^2(\sigma, \bar{\tau}) &= \cos(m\bar{\tau})\theta_0^2 - \sin(m\bar{\tau})\Pi\theta_0^1 + \\ &+ \frac{1}{\sqrt{2}}\sum_{n\neq 0}\left\{\theta_n^2 \mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\bar{\tau} + 2n\sigma)} - \mathrm{i}\Pi\theta_n^1 \mathrm{sign}(n)\mathrm{e}^{-\mathrm{i}(\mathrm{sign}(n)m\bar{\tau} - 2n\sigma)}\right\}, \quad (4.8)\end{aligned}$$

again a perfect match with (3.7, 3.8). We can even go one step further and deduce the tensionless supersymmetric lagrangian from the tensile one:[5] The bosonic part of the light-cone lagrangian [29] reads

$$\tilde{\mathcal{L}}_B = \partial_\tau \tilde{X}^I \partial_\tau \tilde{X}^I - \partial_\sigma \tilde{X}^I \partial_\sigma \tilde{X}^I - \tilde{m}^2 \tilde{X}_I^2 . \quad (4.9)$$

Applying the rescalings eq. (4.5), we find

$$\tilde{\mathcal{L}}_B = \frac{1}{\lambda}\left(\partial_{\bar{\tau}} X^I \partial_{\bar{\tau}} X^I - \lambda^2 \partial_\sigma X^I \partial_\sigma X^I - m^2 X_I^2\right). \quad (4.10)$$

After identifying $\mathcal{L}_B = \lambda\tilde{\mathcal{L}}_B$ (coming from the combined scaling of $\tau$ which leaves the action invariant), we can take the limit $\lambda \to 0$ on $\mathcal{L}_B$ to obtain

$$\mathcal{L}_B = \dot{X}_I^2 - m^2 X_I^2 . \quad (4.11)$$

If we apply this same procedure to the fermionic part of the lagrangian, we derive

$$\begin{aligned}\tilde{\mathcal{L}}_F &= 2\mathrm{i}\left(\tilde{\theta}^1\bar{\gamma}^-(\partial_\tau + \partial_\sigma)\tilde{\theta}^1 + \tilde{\theta}^2\bar{\gamma}^-(\partial_\tau - \partial_\sigma)\tilde{\theta}^2 - 2\tilde{m}\tilde{\theta}^1\bar{\gamma}^-\Pi\tilde{\theta}^2\right)\\ &= \frac{2\mathrm{i}}{\lambda}\left(\theta^1\bar{\gamma}^-(\partial_{\bar{\tau}} + \lambda\partial_\sigma)\theta^1 + \theta^2\bar{\gamma}^-(\partial_{\bar{\tau}} - \lambda\partial_\sigma)\theta^2 - 2m\theta^1\bar{\gamma}^-\Pi\theta^2\right). \quad (4.12)\end{aligned}$$

With $\mathcal{L}_F = \lambda\tilde{\mathcal{L}}_F$, the limit results in:

$$\mathcal{L}_F = 2\mathrm{i}\left(\theta^1\bar{\gamma}^-\partial_{\bar{\tau}}\theta^1 + \theta^2\bar{\gamma}^-\partial_{\bar{\tau}}\theta^2 - 2m\theta^1\bar{\gamma}^-\Pi\theta^2\right). \quad (4.13)$$

But then, $\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F$ is the light-cone lagrangian we arrived at in eq. (2.8).

[5]For the bosonic part of the lagrangian, similar considerations have been carried out in the context of string field theory [35].

We now take a closer look at the behavior of the commutation relations of the tensile quantum theory in this limit:

$$\begin{aligned} [\tilde{p}_0^I, \tilde{x}_0^J] &= -\frac{\mathrm{i}}{2\pi}\delta^{IJ} \longrightarrow [p_0^I, x_0^J] = -\frac{\mathrm{i}}{2\pi}\delta^{IJ} \\ [\tilde{\alpha}_n^{\mathcal{I}I}, \tilde{\alpha}_{n'}^{\mathcal{J}J}] &= \frac{1}{4\pi}\tilde{\omega}_n\delta_{n+n',0}\delta^{IJ}\delta^{\mathcal{I}\mathcal{J}} \longrightarrow [\alpha_n^{\mathcal{I}I}, \alpha_{n'}^{\mathcal{J}J}] = \frac{m}{4\pi}\mathrm{sign}(n)\delta_{n+n',0}\delta^{IJ}\delta^{\mathcal{I}\mathcal{J}} \\ \{\tilde{\theta}_n^{\mathcal{I}\alpha}, \tilde{\theta}_{n'}^{\mathcal{J}\beta}\} &= \frac{1}{8\pi}\left(\gamma^+\right)^{\alpha\beta}\delta_{n+n',0}\delta^{\mathcal{I}\mathcal{J}} \longrightarrow \{\theta_n^{\mathcal{I}\alpha}, \theta_{n'}^{\mathcal{J}\beta}\} = \frac{1}{8\pi}\left(\gamma^+\right)^{\alpha\beta}\delta_{n+n',0}\delta^{\mathcal{I}\mathcal{J}}\,. \end{aligned} \tag{4.14}$$

This is in perfect agreement with (3.15), (3.16) and (3.18). By glancing at the spectrum of the quantized theory, we learn that this continues in a straight-forward way, which shows that tensionless string theory in the pp-wave background eq. (2.1) is a well-defined limit of ordinary string theory in this context. In flat space, as a contrast, tensionless string theory can be defined by quantizing the classical tensionless lagrangian, but it seems very difficult to obtain it from the quantized theory.

## 5. Homogeneous plane waves

It is an interesting question whether the presented considerations extend to a more general class of plane wave backgrounds or if they are an intrinsic property of the pp-wave background eq. (2.1). For this purpose, this section treats the case of a homogeneous plane wave. The most general smooth such background can be described by a metric containing two constant matrices $k_{IJ}$ and $f_{IJ}$ [32]:

$$\mathrm{d}s^2 = 2\mathrm{d}x^+\mathrm{d}x^- + k_{IJ}x^Ix^J\mathrm{d}x^+\mathrm{d}x^+ + 2f_{IJ}x^I\mathrm{d}x^J\mathrm{d}x^+ + \mathrm{d}x^I\mathrm{d}x_I\,. \tag{5.1}$$

Obviously, in general, such a background is no longer maximally supersymmetric. In this section, we allow for an arbitrary dimension $D$, such that indices $I, J, K$ denote $D-2$ transverse coordinates. Additionally, we allow for a $B$-field given by the $D-2$ non-vanishing components $B_{I+} = h_{IJ}x^J$. Not surprisingly, in $D = 10$ dimensions, and for $h, f = 0$ and $k_{IJ} = -\mu^2\delta_{IJ}$, we get back eq. (2.1). By a rotation of the transverse coordinates, $k$ can be chosen to be diagonal: $k_{IJ} = k_I\delta_{IJ}$. The resulting string theory is an integrable model and was solved by Blau et. al. [33] via a frequency base ansatz:

$$X^I(\sigma,\tau) = \sum_{n=-\infty}^{\infty} X_n^I(\tau)\mathrm{e}^{2\mathrm{i}n\sigma}\,, \qquad X_n^I(\tau) = \sum_{\ell=1}^{2D}\xi_{n\ell}\,a_{n\ell}^I\,\mathrm{e}^{\mathrm{i}\omega_{n\ell}\tau}. \tag{5.2}$$

In the quantized theory, $\xi_{n\ell}$ will become the oscillator modes. The $a_{n\ell}^I$ are vectors given by the eigen directions of the following matrix:

$$M_{IJ}(\omega, n) = (\omega^2 + k_I - 4T^2n^2)\delta_{IJ} + 2\mathrm{i}\omega f_{IJ} + 4\mathrm{i}Tnh_{IJ}\,. \tag{5.3}$$

The allowed frequencies are then determined by $\det M(\omega_{n\ell}, n) = 0$ while the eigen directions are given by $M_{IJ}(\omega_{n\ell}, n)a_{n\ell}^I = 0$. It is easy to see that for $h, f = 0$ and constant $k_I = -\mu^2$, one obtains $\omega_n^2 - \mu^2 - 4T^2n^2 = 0$ or - equivalently - $\omega_{n\pm} = \pm\sqrt{\mu^2 + 4T^2n^2}$ as



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expected. We previously obtained the frequencies in the tensionless case by letting $T \to 0$ directly in the corresponding expression. Looking at eq. (5.3), we find that we may do the same here and we again find that the frequencies become degenerate and equal to the frequencies for $n = 0$:

$$\omega_{n\ell} \to \omega_\ell \,, \qquad \det M(\omega_\ell) \equiv \det M(\omega_\ell, 0) = 0 \,. \tag{5.4}$$

From this short remark, we learn that the huge energy degeneration in the space of states is not a special property of the previously discussed maximally supersymmetric pp-wave background but occurs also for more general plane waves. It seems plausible that this result still holds after going over to the quantum theory here as well.

## 6. Conclusion

We have presented the quantized theory of a tensionless superstring in the the constant plane wave background. This theory is a limit of the quantized tensile superstring. In particular, this implies that the tensionless limit can be consistently obtained in the quantized theory of tensile strings in such pp-waves. We find this an interesting property of these backgrounds that seems to be related to the existence of a dimensionful parameter. As discussed in a series of articles in the past, this seems not to be the case in flat space.

Since the string tension and background mass term always come in the combination $m/T$, we conclude that a tensionless limit for fixed $m$ is equivalent to keeping the tension fixed and letting $m \to \infty$, corresponding to an infinitely curved space.

Interesting open questions prompted by our investigation are those related to the BMN limit discussed above, a generalization to similar backgrounds such as the homogeneous or even more general plane waves, see, e.g. [36], and (again) the relation to higher spin theories.


## Acknowledgments

The authors are grateful to I. Bakas, B. Sundborg, P. Sundell and A. Zheltukhin for illuminating discussions and helpful suggestions. We would also like to thank the referee for useful comments. The research of U.L. is supported in part by VR grant 650-1998368.


## A. Notation

The chiral representation for the $32 \times 32$ Dirac matrices $\Gamma^\mu$ are given by the off-diagonal $16 \times 16$ blocks $\gamma^\mu$'s of the $\Gamma^\mu$'s,

$$\Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \bar\gamma^\mu & 0 \end{pmatrix} . \tag{A.1}$$

The $\gamma^\mu$ satisfy $\bar\gamma^\mu = \gamma^\mu_{\alpha\beta}$ and

$$\gamma^\mu \bar\gamma^\nu + \gamma^\nu \bar\gamma^\mu = 2\eta^{\mu\nu} \,. \tag{A.2}$$

| | |
|---|---|
| $\mu, \nu = 0, 1, \ldots, 9$ | SO(9,1) vector indices |
| $I, J = 1, \ldots, 8$ | SO(8) vector indices |
| $i, j = 1, \ldots, 4$ | SO(4) vector indices |
| $i', j' = 5, \ldots, 8$ | SO′(4) vector indices |
| $\alpha, \beta, \gamma = 1, \ldots, 16$ | SO(9,1) spinor indices |
| $\mathcal{I}, \mathcal{J} = 1, 2$ | labels for the two sets of oscillators |
| $a, b = 0, 1 = \tau, \sigma$ | 2D world-sheet coordinate indices |

**Table 3:** Conventions for the indices.

The $\gamma^\mu$ matrices are real and symmetric. The fermionic mass operators $\Pi$ and $\bar{\Pi}$ satisfying $\Pi^2 = 1$ are given by

$$\begin{aligned} \Pi^\alpha{}_\beta &\equiv (\gamma^1\bar{\gamma}^2\gamma^3\bar{\gamma}^4)^\alpha{}_\beta\,, \qquad & \Pi'^\alpha{}_\beta &\equiv (\gamma^5\bar{\gamma}^6\gamma^7\bar{\gamma}^8)^\alpha{}_\beta \\ \bar{\Pi}^\alpha{}_\beta &\equiv (\bar{\gamma}^1\gamma^2\bar{\gamma}^3\gamma^4)^\alpha{}_\beta\,, \qquad & \bar{\Pi}'^\alpha{}_\beta &\equiv (\bar{\gamma}^5\gamma^6\bar{\gamma}^7\gamma^8)^\alpha{}_\beta \end{aligned} \tag{A.3}$$

The identity $(\gamma^0\bar{\gamma}^9)^2 = 1$ implies

$$\gamma^0\bar{\gamma}^9\gamma^\pm = \pm\gamma^\pm\,, \qquad \bar{\gamma}^\pm\gamma^0\bar{\gamma}^9 = \mp\bar{\gamma}^\pm\,, \tag{A.4}$$

allowing us to define the $\gamma^\pm$, $\bar{\gamma}^\pm$ as eigenmatrices of $\gamma^0\bar{\gamma}^9$. For a more detailed prescription we refer to [28, 29].

## B. The SUSY-algebra

The symmetries of the supersymmetric action can be found following the lines in [28]: The $\kappa$-symmetry fixed lagrangian eq. (2.6) in complex $\theta, \bar{\theta}$ notation, where $\theta = \theta^1 + \mathrm{i}\theta^2$ reads

$$\mathcal{L} = \left(2\dot{X}^+\dot{X}^- - X_I^2(\dot{X}^+)^2 + \dot{X}_I^2\right) + 2\mathrm{i}\dot{X}^+\left(\bar{\theta}\bar{\gamma}^-\dot{\theta} + \theta\bar{\gamma}^-\dot{\bar{\theta}} + 2\mathrm{i}\dot{X}^+\bar{\theta}\bar{\gamma}^-\Pi\theta\right). \tag{B.1}$$

Here, $v$ and $\mu$ are gauge fixed to 1, but can be introduced as in the previous sections. Since we did not change the background in comparison to Metsaev, the (infinitesimal) transformations giving the symmetries of the lagrangian should be the same. Indeed, translations in the $X^-$-plane ($\delta X^- = \epsilon^-$) give rise to a conserved current $\mathcal{P}^{+a} = 2\dot{X}^+\delta^{a0}$, where $a = (0,1)$ are the directions on the world-sheet. It is then easy to check that the conserved kinematical currents for our lagrangian are given by:

$$\mathcal{P}^{+0} = 2\dot{X}^+ \tag{B.2}$$
$$\mathcal{P}^{-0} = 2(\dot{X}^- + \mathrm{i}(\bar{\theta}\bar{\gamma}^-\dot{\theta} + \theta\bar{\gamma}^-\dot{\bar{\theta}})) - (X_a^2 + 4\bar{\theta}\bar{\gamma}^-\Pi\theta)\mathcal{P}^{+0} \tag{B.3}$$
$$\mathcal{P}^{I0} = 2\cos(X^+)\dot{X}^I + \sin(X^+)X^I\mathcal{P}^{+0} \tag{B.4}$$
$$\mathcal{J}^{+I0} = 2\sin(X^+)\dot{X}^I - \cos(X^+)X^I\mathcal{P}^{+0} \tag{B.5}$$
$$\mathcal{J}^{ij0} = 2(X^i\dot{X}^j - X^j\dot{X}^i) - \mathrm{i}\bar{\theta}\bar{\gamma}^-\gamma^{ij}\theta\mathcal{P}^{+0} \tag{B.6}$$
$$\mathcal{J}^{i'j'0} = 2(X^{i'}\dot{X}^{j'} - X^{j'}\dot{X}^{i'}) - \mathrm{i}\bar{\theta}\bar{\gamma}^-\gamma^{i'j'}\theta\mathcal{P}^{+0} \tag{B.7}$$
$$\mathcal{Q}^{+0} = 2\bar{\gamma}^-\mathrm{e}^{\mathrm{i}X^+\Pi}\mathcal{P}^{+0}\theta \tag{B.8}$$
$$\bar{\mathcal{Q}}^{+0} = 2\bar{\gamma}^-\mathrm{e}^{-\mathrm{i}X^+\Pi}\mathcal{P}^{+0}\bar{\theta}\,. \tag{B.9}$$



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If we compare these currents to those obtained for the tensile case [28], we see that the only difference apart from the fact that only one of the two world-sheet directions contribute, is the absence of terms proportional to $\epsilon^{ab}$. These terms are connected to the WZW-term in eq. (2.2) which vanishes in the tensionless limit. This has, however, no effect on the conserved charges generating the transformations. In light-cone gauge, these charges are:

$$P^+ = p^+ \tag{B.10}$$

$$P^I = \int \cos(X^+)\mathcal{P}^I + \sin(X^+)X^I p^+ \tag{B.11}$$

$$J^{+I} = \int \sin(X^+)\mathcal{P}^I - \cos(X^+)X^I p^+ \tag{B.12}$$

$$Q^+ = \int 2p^+\bar{\gamma}^- \mathrm{e}^{\mathrm{i}X^+\Pi}\theta \tag{B.13}$$

$$\bar{Q}^+ = \int 2p^+\bar{\gamma}^- \mathrm{e}^{-\mathrm{i}X^+\Pi}\bar{\theta}\,. \tag{B.14}$$

Using the SUSY-algebra, we then calculate the dynamical currents and charges $\mathcal{Q}^{-0}$ and $Q^-$, and obtain:

$$\mathcal{Q}^{-0} = 2\mathcal{P}^I\bar{\gamma}^I\theta + 2\mathrm{i}p^+X^I\bar{\gamma}^I\Pi\theta \qquad \text{and} \qquad \bar{\mathcal{Q}}^{-0} = 2\mathcal{P}^I\bar{\gamma}^I\bar{\theta} - 2\mathrm{i}p^+X^I\bar{\gamma}^I\Pi\bar{\theta}\,. \tag{B.15}$$

Since

$$\partial_0\mathcal{Q}^{-0} = 2p^+\left(\ddot{X}^I + m^2X^I\right)\bar{\gamma}^I\theta\,, \tag{B.16}$$

where we introduced the parameter $m$, we find that $\mathcal{Q}^{-0}$ is time independent using eq. (3.1). The corresponding charge is then derived by integration over $\sigma$, just as in the above cases.

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